Abstract: This paper presents a method to determine the natural frequencies of transversal bending vibrations of beams and plates by means of time-history analysis. As excitation source the sound pressure produced by a speaker was used; it permits an accurate control of the exciting frequency. The method consists in exciting the beam/plate around the resonance frequency analytically determined and find the frequency value that produces the highest amplitudes of the stabilized system, without inducing the beat phenomena. By departing from that frequency in sense of increasing or decreasing, the amplitude is diminished and signal envelope becomes modulated in amplitude. It can be used in application where contact between the system and the exciter is not possible or permitted.

Keywords: - beam, vibration, natural frequency, excitation, sound pressure

1. INTRODUCTION

Information regarding the natural frequency of mechanical systems is often requested, in order to avoid resonance, [1]-[3], or to get information about its structural health [4]-[5]. There are different ways to find these frequencies, see [1]-[2] and [6], considering different methodologies and algorithms. Sometimes the structure-exciter contact is not permitted or even impossible, therefore exciting techniques have to be involved. The paper present a method to determine the natural frequencies, based on an exciting system composed by an amplifier and a loudspeaker, crating a sound pressure in the frequency range close to the resonant one.

2. THEORETICAL CONSIDERATIONS

In this paper the method is applied to a straight aluminum beam, clamped at both ends, with the following dimensions: length $L = 1.5$ m, width $B = 20$ mm and height $H = 2$ mm. For this data result $A = 40\cdot10^{-6}$ m$^2$ and $I_z = 13.333\cdot10^{-12}$ m$^4$. The Young’s modulus of aluminum is $E = 70000$ N/mm$^2$ and the density $\rho = 2700$ kg/m$^3$. With these values, we calculated the first five natural frequencies of the aluminum beam using the relation:

$$f_n = \frac{(\alpha_n)^2}{2\pi} \sqrt{\frac{EI_z}{\rho AL^4}} \tag{1}$$

where $n = 1, 2, 3, 4, 5$ are the mode numbers. The obtained results are shown in Table 1 and the representation of the modes in Fig 1.

Table 1 Natural frequency values for the analyzed beam

<table>
<thead>
<tr>
<th>Mode $n$</th>
<th>Roots $\alpha_n L$</th>
<th>Frequency $f_n$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.730041</td>
<td>46</td>
</tr>
<tr>
<td>2</td>
<td>7.853205</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>10.995608</td>
<td>251</td>
</tr>
<tr>
<td>4</td>
<td>14.137165</td>
<td>415</td>
</tr>
<tr>
<td>5</td>
<td>17.278760</td>
<td>620</td>
</tr>
</tbody>
</table>

Figure 1. The shapes and positions of the nodal point’s bumper recessed at both ends
3. THE EXPERIMENTAL STAND

The experimental stand presented in Fig 2(a) is composed by a wooden plate on which are fixed 2 modules embedding the bar, providing a clamped-clamped condition, for the beam length of 1 m. For the experiment we used an aluminum bar, Fig. 2(b), of rectangular shape, with following dimensions: length: 1.5 m, width: 20 mm and height: 2 mm.

![Image 1](image1.png)

**Figure 2.** The experimental stand (a) with the aluminum Beam (b) and the speaker used to generate vibrations (c)

To generate the necessary vibration we used a speaker, Fig. 2(c), for low frequency (bass), having a diameter of 165 mm, impedance of 4 ohms and output of 50 W.

In order to control the signal sent to the speaker we use the program NCH Tone Generator, the interface being presented in Fig. 3.

![Image 2](image2.png)

**Figure 3.** The main window of NCH Tone Generator

The program can generate frequencies between 1÷22.000 Hz, allowing choosing the signal shape: sine, triangle, saw tooth, pulse. Also the sample rate can be adjusted between 6000 and 192000 Hz, together with other features as: signal duration, variable amplitude, linear or logarithmic sweep.

As acquisition system we use a Kistler accelerometer 8772A5 attached to a 4 channels NI 9234 module and an NI cDAQ-9172 chassis. The signal is converted and transmitted via a USB connection to a Toshiba laptop, where they are stored and processed. The whole system is presented in Fig. 4.

![Image 3](image3.png)

**Figure 4.** The acquisition system

The data acquisition as well as the processing is made by use of Virtual Instruments developed in the LabVIEW program [7]. One instrument is used to acquire and store the signal; another, presented in Fig. 5, is used for reading the stored signal and determines its time history and the corresponding natural frequencies. The acquisition of vibration signals can be done in different parts of the Beam by connecting one or more accelerometers.

![Image 4](image4.png)

**Figure 5.** LabVIEW interface for reading/processing acquired data from the bar

The signal acquisition form the aluminum Beam was done using the assembly shown in Fig. 3(a). The speaker that generates vibration was mounted with the membrane directly face to the front of the aluminum bar, at a distance of 0.1 m, so that vibrations are transmitted without significant energy loss.

The accelerometer location was chosen on a position along the beam that assures the highest displacement (i.e. acceleration).
4. METHODOLOGY AND RESULTS

The chart shown in Fig. 6 depicts the beam’s natural frequencies for the first three transversal weak-axis bending vibration modes, of 46 Hz, 128 Hz and 251 Hz. The signal was generated by hammer hitting; to emphasize a certain natural frequency the resonance of it is looked for. This can precisely be made by a non-contact excitation, produced for instance using a speaker or a white/pink noise source. In our experiments we used a speaker with controllable frequency, as shown in the previous section.

![Figure 6. Natural frequencies of the aluminum Beam](image)

We have established further acquisition around 128 Hz, which is one of the beam frequencies, with the following results.

![Figure 7. Beam’s response to an excitation of 120 Hz](image)

From the beam response at 120 Hz, shown in Fig. 7, it is noted that in the beginning of the excitation the beam vibrates with high amplitude, being afterwards stabilized at lower amplitude. The envelope, resulting as a superposition of the two signals, is modulated in amplitude with high frequency. At 121 Hz, Fig. 8, the beam’s behavior is similar to that of 120 Hz, with small differences of oscillation amplitude and changing the shape of the signal envelope in sense of decreasing the modulation frequency.

At frequencies of 122 and 123 Hz, Fig. 9 and 10, we observe an increase of the oscillation amplitude, associated with an increase of the modulation period.

At 124 Hz Fig. 11 we observe a significant increase in amplitude.

![Figure 8. Beam response to an excitation of 121 Hz](image)

![Figure 9. Beam response to an excitation of 122 Hz](image)

![Figure 10. Beam’s response to an excitation of 123 Hz](image)

![Figure 11. Beam’s response to an excitation of 124 Hz](image)

At 125 Hz Fig. 12 we observe a significant increase in amplitude together with a rapid signal stabilization. It shows that the frequency range is close to the resonance domain.

Exciting the beam with 126 Hz results a time-history signal like that presented Fig. 13. It can be observed that the vibration amplitude strongly increases and attend a maximum maintained throughout the whole excitation duration.
Thus, the beam is at resonance, the beats phenomena disappears. The experimentally determined frequency of 126 Hz deviates from the calculated value of 128 Hz with 1.56 %, which can be justified by the mass added to the system due to the use of the accelerometer.

Fig. 14 present the time-history for the beam excited at 127 Hz; it is not longer in resonance since the amplitude is stabilized at a lower value and the beat phenomena occurs again. The excitation of 128 Hz, Fig. 15, produce similar effects to that of 127 Hz, but the vibration amplitude is smaller, meaning that the exciting frequency depart from the resonance frequency.

For higher excitation frequencies the beat phenomena is stronger present, with increasing frequency of the amplitude modulation, showing that the frequencies depart from the resonance.

5. CONCLUSIONS

The time-history of vibration signals acquired from beams with controlled excitation permit finding of natural frequencies with high accuracy. The used experimental stand is simple, involving a speaker, but the results can be improved by using a white/pink noise generator. The resonance is indicated by the disappearance of the beat phenomena. Further researches will focus on developing an automated method to determine a large series of frequencies at once.

REFERENCES