
On the Spatial Characteristics of a Circular Piston

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Abstract: - In this paper a theoretical overview of existing and well known mathematical techniques for calculating the sound pressure level (SPL) created by a circular piston in its near and far field is presented. A unified approach for explaining the acoustic transducers' spatial characteristics is proposed. The polar responses and the frequency responses of given circular pistons are modeled and visualized by means of Matlab®-based scripts. Finally some experimental data are presented. Conclusions with practical value have been made. A few topics for future works are considered.

Keywords: - Polar response, Frequency response, Near field, Far field, Matlab®.

1. INTRODUCTION

It is well known that the transducer's sound field has two distinct zones. The first one, called the Near Field, is in the space directly in front of the transducer, while the second one, called Far Field, is the space beyond the Near Field where the sound field pressure gradually drops to zero (i.e. equal to the atmospheric pressure) [1]. The sound pressure in the far field is discussed in detail by R. Aarts and A. Janssen [2, 3, 4, 5]. The sound pressure in the near field however [6, 7, 8] is object of far less examinations. In experimental vibro-acoustics a third area is identified too - very near field [9, 10, 11, 12].

In this paper a theoretical overview of existing and well known mathematical techniques for calculating the sound pressure level (SPL) created by a circular piston in its near and far field is presented. A unified approach for explaining the acoustic transducers' spatial characteristics is proposed. The polar responses and the frequency responses of given circular pistons are modeled and visualized by means of Matlab®-based scripts. Finally some experimental data are presented.

In order to understand the sound field created by a transducer it is necessary to examine its spatial characteristics. The latter depends on constant and variables parameters. Parameters independent of the source of acoustic waves (i.e. the transducer), but dependent on the environment are:

- speed of sound - c_0 ;
- atmospheric pressure - p_s ;
- particles velocity - v_m ;
- density of the environment - ρ_s ;

- temperature - t° ;
- etc.

Parameters related to the source of acoustic waves are:

- transducer's radius - a ;
- directivity angle - θ° ;
- etc.

There are two basic variables of the sound field – sound pressure p_a and particles velocity v_m . Sound pressure is the physical quantity (parameter) that gives detailed information about the sound field. Hence, in order to understand the physics of the sound field it is necessary to calculate the SPL (for each point of the space) created by transducers with different geometry (rectangular, circular, elliptical etc.).

2. THEORETICAL BACKGROUND

The object of this study is a circular piston. Due to its symmetry with respect to z axis (Fig. 1) one can assume that its polar response (pattern) will be one with rotational symmetry. To define that symmetry it is sufficient to determine the SPL at distance r on a plane whose normal is parallel to the z axis.

The total acoustic pressure created by the circular piston is [13, 14]:

$$p(\theta, f, r, t) = j\rho_s c_0 \frac{v_m}{\lambda} \int_Q \frac{1}{y} e^{j(\omega t - ky)} dQ, \quad (1)$$

where: $k = 2\pi f / c_0$ is wave number;
 $\lambda = c_0 / f$ – wave length;

y – the distance between the elementary section dQ and point A (Fig. 1).

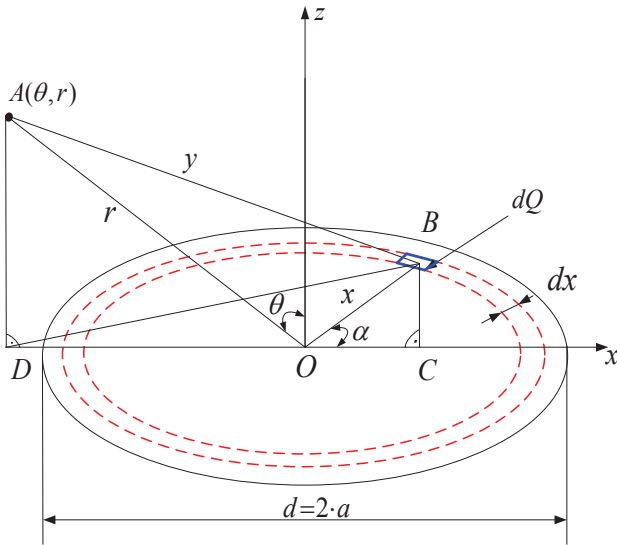


Figure 1. Geometry of circular piston directivity.

The acoustic pressure amplitude for the far field is [13]:

$$p_a(\theta, f, r) = \frac{\rho_s f v_m}{r} e^{j\omega t} \cdot \int_0^a x dx \int_0^{2\pi} e^{-j \frac{2\pi f}{c_0} (r + x \sin \theta \cos \alpha)} d\alpha. \quad (2)$$

For the far field by Eq. (1) and assuming that $y = r$ and $r \gg a$ the polar response (directivity function) of the circular piston is [13, 14, 15]:

$$G(\theta, f) = \frac{2J_1\left(\frac{2\pi f a \sin \theta}{c_0}\right)}{\frac{2\pi f a \sin \theta}{c_0}}. \quad (3)$$

where $J_1\left(\frac{2\pi f a \sin \theta}{c_0}\right)$ is Bessel function of the first kind of the argument $\left(\frac{2\pi f a \sin \theta}{c_0}\right)$.

For the near field the acoustic pressure amplitude on the axis of the piston - Eq. (1) can be simplified [14, 16]:

$$p_a(0, f, r) = 2\rho_s c_0 v_m \cdot \left| \sin \left\{ \frac{\pi f}{c_0} \left[\sqrt{r^2 + a^2} - r \right] \right\} \right|. \quad (4)$$

where r is the distance between the transducer and point A .

One should be aware that in Eqs. 1, 2 and 4 the parameters of the excitation signal (e.g. amplitude, frequency content, etc.) and the transducer sensitivity are not taken into account.

Study of Eq. (4) reveals that the axial acoustic pressure extremes occur at [14, 16]:

$$\frac{\pi f}{c_0} \left[\sqrt{r^2 + a^2} - r \right] = m\pi, \quad m = 0, 1, 2, \dots \quad (5)$$

Furthermore, zeros (minimal values) are to be expected in the SPL for [14]:

$$r = \frac{a^2}{2\lambda} - \frac{\lambda}{2} \quad (6)$$

Study of Eq. (2) reveals that the acoustic pressure amplitude $p_a(\theta, f, r)$ at given point $A(\theta, r)$ (Fig. 1) depends on the signal frequency f . Therefore a 4D plot is to be needed (Fig. 2) for visualizing Eq. (2).

Analysis of Fig. 2 needs to be done in sets of slices. Every slice consists of different frequency response or polar response families. Some of these sets can be easily and unambiguously understood and interpreted. There is no conventional interpretation for the others. Considering Fig. 2 one can differ:

- a horizontal set of polar response families $p_a(\theta, r)$ for different frequencies f . The $p_a(\theta, r)$ itself is a family of polar responses $p_a(\theta)$ for different distances r (it must be taken into account that the SPL decreases inversely with the distance - $1/r$ [17, 18]);

- a lateral set of frequency response families $p_a(f, r)$ for different angles θ . The $p_a(f, r)$ itself is a family of frequency responses $p_a(f)$ for different distances r (the SPL decreases inversely with the distance - $1/r$ [17, 18]);

- a frontal set which represents the dependence $p_a(\theta, f)$ for different distances r .

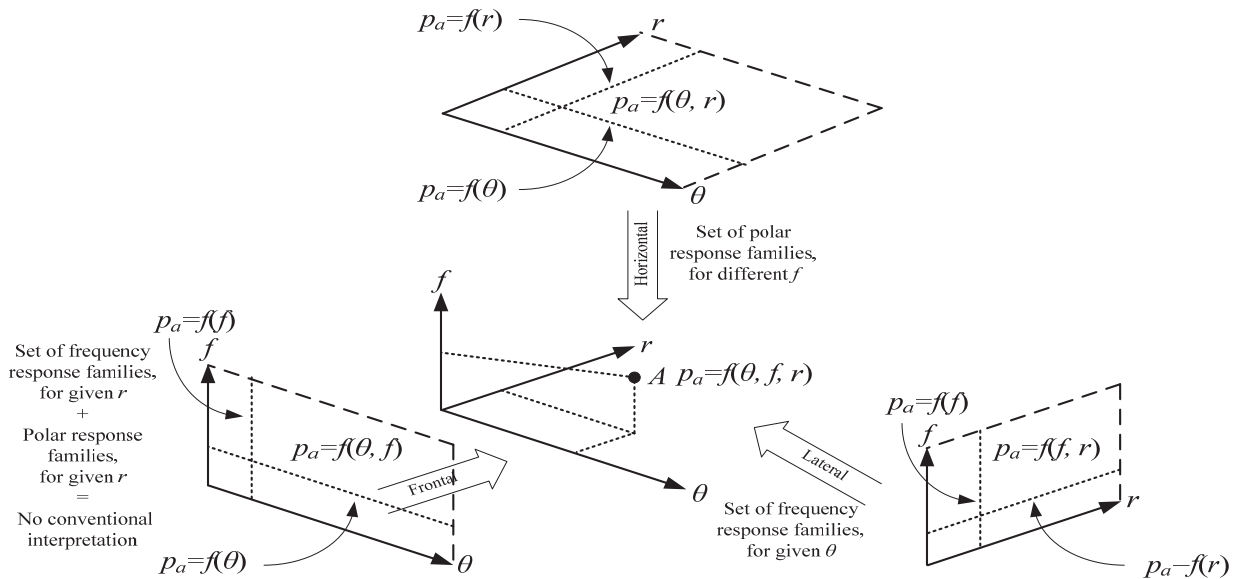


Figure 2. Spatial characteristic of a circular piston (for any angle, frequency and distance).

The $p_a(\theta, f)$ itself contains interlaced frequency response and polar response families. There isn't conventional interpretation for this case.

3. GRAPHICAL REPRESENTATION

In Fig. 3 the acoustic pressure amplitude $p_a(\theta, f, r)$, in the far field, created by the transducer with radius $a = 0.08m$ is presented (similarly to Fig. 2).

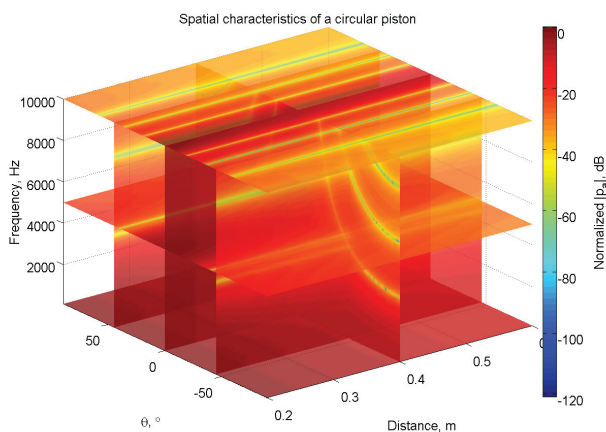


Figure 3. Spatial characteristic of a circular piston in the far field.

The 4D plot consists of frequency response families (at different angles) and polar response families (at different frequencies) for the far field ($r > 0.2m$).

The figure could be explored in details on: <https://www.mathworks.com/matlabcentral/fileexchange/51136-exploration-of-the-spatial-characteristics-of-a-circular-piston-with-matlab-implementation>.

3.1. Polar response in the far and in the near field

One can model and visualize the polar response of a given transducer in the far field by Eq. (3).

Fig. 4 shows a sectional view (slice) in polar coordinates of the 4D plot (Fig. 3). The slice is a transducer's polar responses at different distances for constant frequency $f = 5899.5 Hz$. The transducer's radius is $a = 0.065 m$.

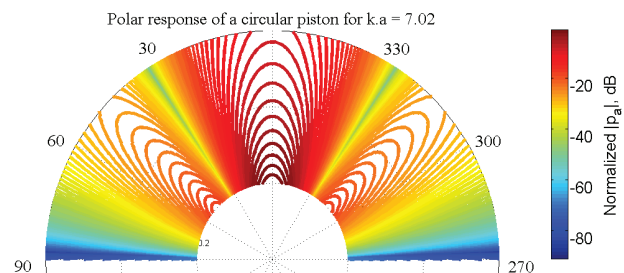


Figure 4. Polar response vs. distance (for $k \cdot a = 7.02$ - second Bessel function root).

Fig. 5 shows a transducer's frequency responses at different distances for constant angle $\theta = 30^\circ$.

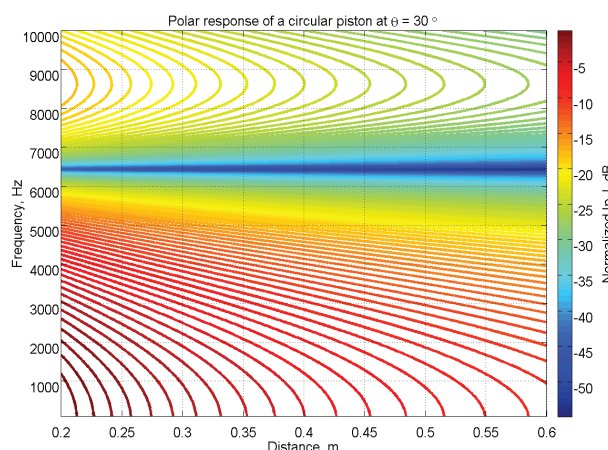


Figure 5. Frequency response vs. distance (for $\theta = 30^\circ$).

3.2. Frequency response in the far and in the near field

For modeling and visualizing the SPL around the transducer a Matlab[®] software is used. This software calculates Eq. (2) and Eq. (4) by means of numerical analysis in order to determine the frequency response of a given transducer.

The results are presented in Fig. 6 (far field) and Fig. 7 (near field). The 3D plots reveal a difference between the SPL calculated by Eq. 2 and the SPL calculated by Eq. 4 in the immediate proximity of the transducer.

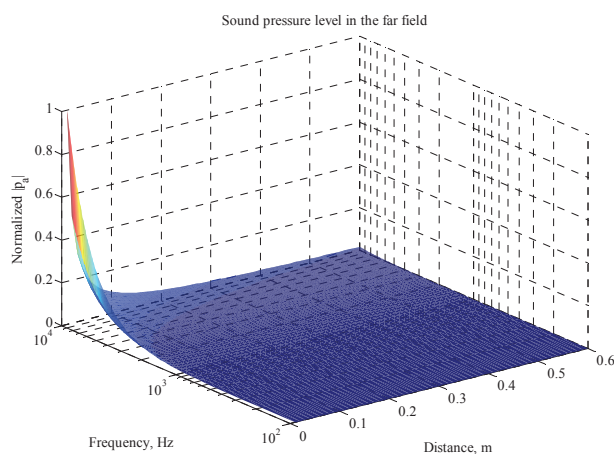


Figure 6. 3D plot of normalized SPL in the far field calculated by Eq. (2) for $\theta = 0^\circ$ (i.e. on-axis).

By using Eq. (6) the SPL created by transducer with radius $a = 0.08m$ at frequency $f = 4500Hz$ will have a minimum at distance $r = 0.004m$. In Fig. 8 and Fig. 9 the frequency response of the transducer at distance $r = 0.004m$ is presented (calculated by Eq. 2 and Eq. 4 respectively).

In Fig. 8 the frequency response of the transducer calculated by Eq. 2 lacks any minimums. On the

other hand the frequency response calculated by Eq. 4 (Fig. 9) falls to a local minimum for different frequencies. At distance $r = 0.004m$ one encounters the first local minimum at frequency $f = 4500Hz$.

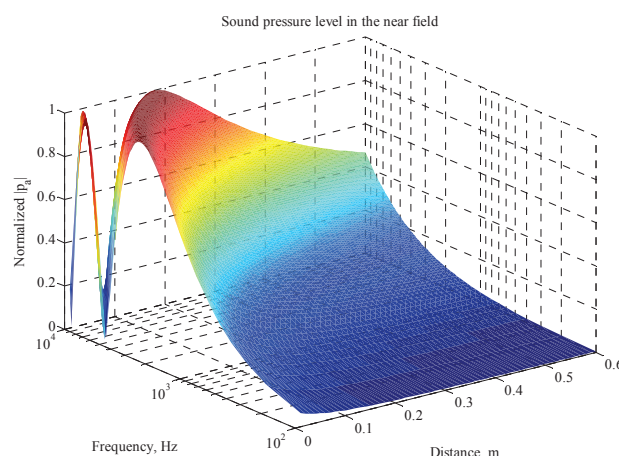


Figure 7. 3D plot of normalized SPL in the near field calculated by Eq. (4) for $\theta = 0^\circ$ (i.e. on-axis).

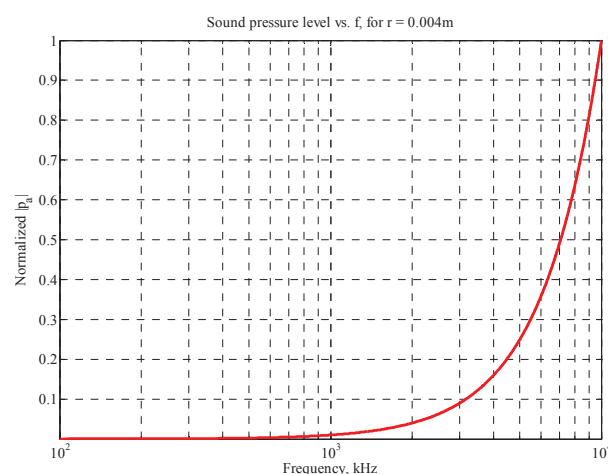


Figure 8. Frequency response calculated by Eq. (2) for $r = 0.004m$.

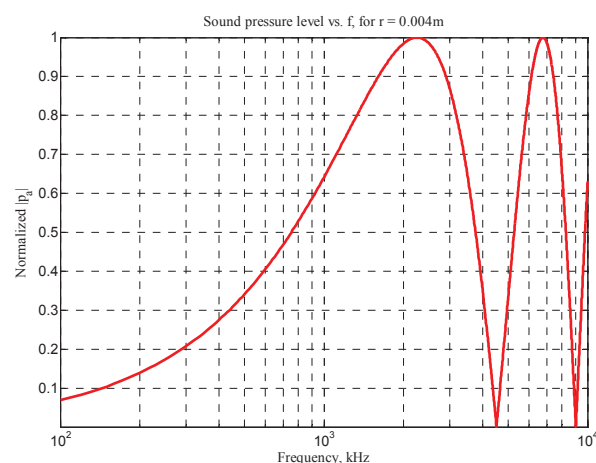


Figure 9. Frequency response calculated by Eq. (4) for $r = 0.004m$.

In Fig. 10 and Fig. 11 the correlation between the SPL and the distance at frequency $f = 4500 \text{ Hz}$ is presented (calculated by Eq. 2 and Eq. 4 respectively).

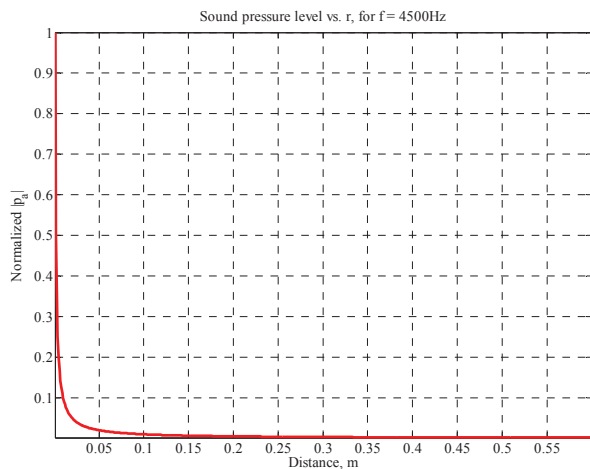


Figure 10. SPL vs. distance (far field) calculated by Eq. (2) for $f = 4500 \text{ Hz}$.

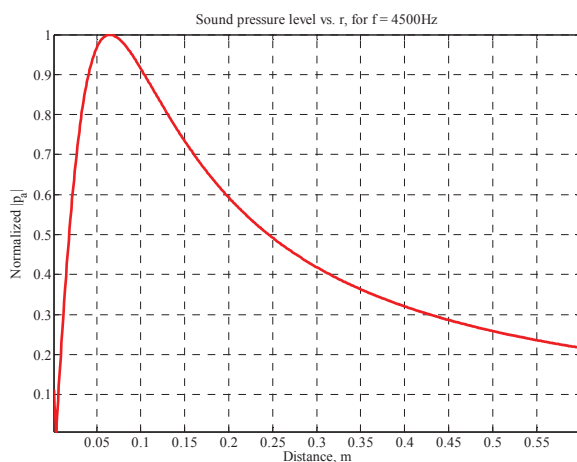


Figure 11. SPL vs. distance (near field) calculated by Eq. (4) for $f = 4500 \text{ Hz}$.

4. EXPERIMENTAL RESULT

4.1. Polar response in the far and in the near field

The practical measurements were carried out with a measurement complex Velleman-PCSGU250, a laptop, a sound-level meter Robotron Präzisions, a microphone MK221 and loudspeakers – VVK 201 (diameter $d = 0.16 \text{ m}$) and JBL (diameter $d = 0.13 \text{ m}$).

In Fig. 12 the black solid curve with stars represents the measured polar response of a JBL loudspeaker at distance $r = 0.6 \text{ m}$ (far field), while in Fig. 13 the curve represents the measured polar response at distance $r = 0.15 \text{ m}$ (near field).

The analysis of the obtained data shows that Eq. 3 is inaccurate in the immediate proximity of the transducer (i.e. near field). This inaccuracy can be explained with the assumption made for the far field $y = r$.

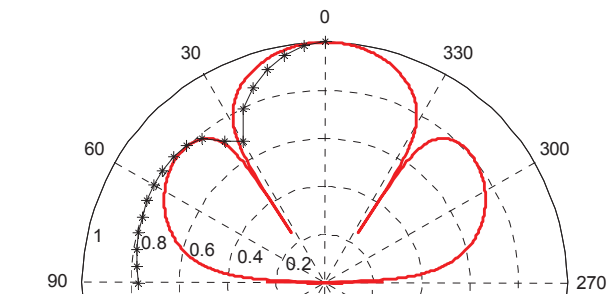


Figure 12. Polar response at $r = 0.6 \text{ m}$ and $k \cdot a = 7.02$.
Solid red curve – Eq. (3);
Black solid curve with stars – measured data ($0^\circ \div 90^\circ$).

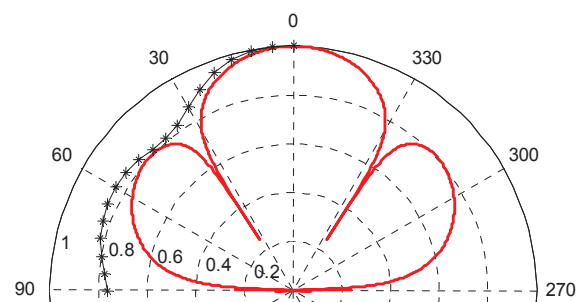


Figure 13. Polar response at $r = 0.15 \text{ m}$ and $k \cdot a = 7.02$.
Solid red curve – Eq. (3);
Black solid curve with stars – measured data ($0^\circ \div 90^\circ$).

4.2. Frequency response in the far and in the near field

Measured frequency response of a loudspeaker VVK 201 at distance $r = 0.004 \text{ m}$ is shown in Fig. 14. The frequency response falls to a local minimum at frequency $f = 4500 \text{ Hz}$ (marked with red circle).

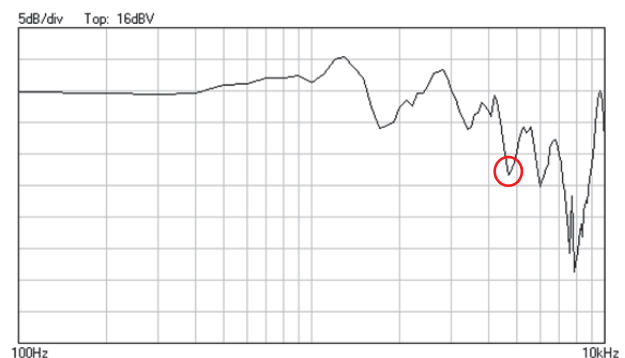


Figure 14. Measured frequency response of a circular piston at $r = 0.004 \text{ m}$.

In Fig. 15 the measured frequency response of a loudspeaker VVK 201 at distance $r = 0.6\text{m}$ (far field) is shown.

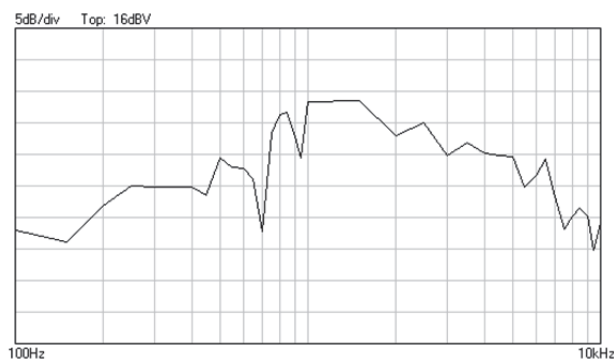


Figure 15. Measured frequency response of a circular piston at $r = 0.6\text{m}$.

5. CONCLUSION

The theoretical overview and experimental data analysis reveal that the proposed unified approach is applicable for explaining and understanding the acoustic transducers' spatial characteristics.

What is of interest is whether there is a distinguishable border between the near and the far field (for a given transducer). Furthermore, is there a suitable criterion for defining that border?

Even though the Rayleigh integral gives the sound pressure in the space around the transducer, is there any other modified and appropriate expression which calculates the sound pressure in both the near and the far field.

In Fig. 14 there are other extremes that are not related to Eq. 4 (i.e. the physics of the near field). In Fig. 15 there aren't any minimums at frequency $f = 4500\text{Hz}$ but there are other extremes at hand. The cause for those extremes is of interest.

These problems will be at focus in future discussions.

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